USN | | | | | | | | | | |

14ELD11

First Semester M.Tech. Degree Examination, June/July 2016
Advanced Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Apply shifted QR algorithm to A and hence find eigen values of $A = \begin{pmatrix} 3 & 1 \\ 1 & 5 \end{pmatrix}$. **(10 Marks)**

b. Find the generalized inverse of,

$$A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{bmatrix}.$$ **(10 Marks)**

2 a. Find the extremals of $\int_1^2 \frac{\sqrt{1+y'^2}}{x} dx$ given that $y(1) = 0$, $y(2) = 1$. **(10 Marks)**

b. Find the path on which a particle in the absence of friction will slide from one point to another in shortest time under the action of gravity. **(10 Marks)**

3 a. Find a function $y(x)$ for which $\int_0^1 x^2 + y'^2 dx$ is a stationary function given that,

$\int_0^1 y^2 dx = 2$, $y(0) = 0$, $y(1) = 0$. **(10 Marks)**

b. Find the distance between a parabola $y = x^2$ and a straight line $x - y = 5$. **(10 Marks)**

4 a. A string is stretched between two points $x = 0$ and $x = l$. The motion is started by displacing the string in the form $u = \sin\left(\frac{\pi x}{l}\right)$, $0 < x < l$ and released from rest at $t = 0$. Find the displacement at any point of string at any time t. **(10 Marks)**

b. Solve the heat conduction equation, $K\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $-\infty < x < \infty$, $t > 0$ given that

$u(x,t) \to 0$ as $x \to \pm\infty$

$u_x(x,t) \to 0$ as $x \to \pm\infty$

and $u(x,0) = f(x)$, $-\infty < x < \infty$. **(10 Marks)**

5 a. Using Laplace transform method solve $u_{xx} = \frac{1}{C^2}u_{tt} - \cos\omega t$, $0 < x < \infty$ given that $u(0,t) = 0$, u is bounded as $x \to \infty$, $u(x,0) = 0$, $u_t(x,0) = 0$, $t > 0$, $0 < x < \infty$. **(10 Marks)**

b. Find the temperature u(x, t) in a semi-infinite rod $0 < x < \infty$, $t > 0$ subject to

$u(x, 0) = 0$ $0 < x < \infty$,

$u_x(0,t) = -u_0$ (a constant) $u(x,t)$ is bounded as $x \to \infty$. **(10 Marks)**

6 a. Solve $u_{xx} + u_{yy} = 0$, $x \geq 0$, $y \geq 0$ given that $u(0,y) = 0$, u & $\frac{\partial u}{\partial x} \to 0$ as x, $y \to \infty$ and $u_y(x,0) = f(x)$. **(10 Marks)**

b. Define harmonic function. If ϕ is a harmonic function in R and $\frac{\partial \phi}{\partial x} = 0$ on ∂R then show that ϕ is constant in $\overline{R}$. **(10 Marks)**

7 a. Use two phase simplex method to minimize $z = 7.5x_1 - 3x_2$, subject to $3x_1 - x_2 - x_3 \geq 3$, $x_1 - x_2 + x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$. **(10 Marks)**

b. Use Lagrange's multipliers method to minimize $z = x_1^2 + x_2^2 + x_3^2$ subject to $x_1 + x_2 + 3x_3 = 2$, $5x_1 + 2x_2 + x_3 = 5$, $x_1, x_2, x_3 \geq 0$. **(10 Marks)**

8 a. Use dual simplex method to solve LPP,

Minimize $z = 2x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$ **(10 Marks)**

b. Use Kuhn-Tucker method to

Maximize $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

Subject to $2x_1 + x_2 \leq 5$, $x_1, x_2 \geq 0$ **(10 Marks)**

* * * * *